647510 **BSc6MatC601x** **Seat No :________**

B.Sc. Semester - 6 (*CBCS*) Examination
May/June-2021 (NEW COURSE)
Graph Theory and Complex Analysis -2(Core)

Time: 1:30 Hours **Marks: 42**

Instructions:

1. Figures to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any three of the following questions.

Q.1(A) Answer any two out of three. (10)

(1) Prove that if a graph has exactly two vertices of odd degree, there must be a path joining these two vertices.

(2) State and prove necessary and sufficient condition for being Euler graph.

(3) Define: Hamiltonian circuit. Prove that number of pendant vertex in binary tree is $\frac{n+1}{2}$.

Q.1(B) Answer any one out of two. (04)

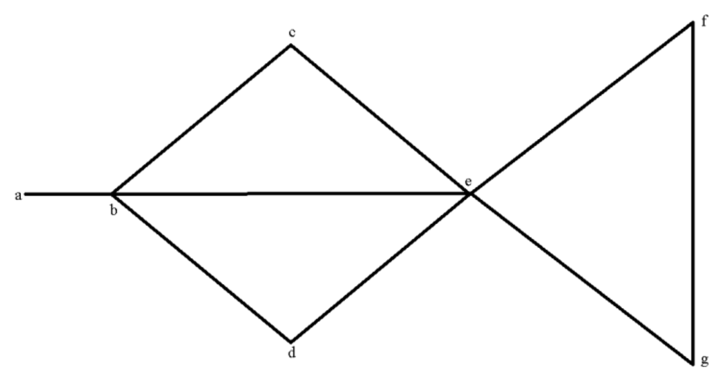
(1) Prove that number of vertices of odd degree in a graph is always even.

(2) Define: Walk, Path, Complete graph, Tree.

Q.2(A) Answer any two out of three. (10)

(1) In usual notation for any simple, connected planer graph prove that $e \geq \frac{3}{2}f$ and $e \leq 3n-6$.

(2) For the following graph G find maximal independent sets.



(3) G is a (4,4) graph where V={a,b,c,d} E={ e_1, e_2, e_3, e_4 } where $\varphi(e_1)$=ab, $\varphi(e_2)$=bc, $\varphi(e_3)$=cd, $\varphi(e_4)$=bd then find

(i) dim(W_T), base of W_T and W_T (ii) dim(W_S), base of W_S and W_S.

Q.2(B) Answer any one out of two. (04)

(1) Define planner graph. Prove that complete graph with 5 vertices is nonplanner.

(2) Prove that a vertex v in a connected graph G is cut vertex if and only if there exist two vertices x and y in G such that every path between x and y passes through v.

Q.3(A) Answer any two out of three. (10)

(1) Show that the set of all bilinear maps under the composition of maps forms a group.

(2) Prove that every Mobious transformation maps circles OR straight lines into circles OR straight lines.

(3) Define: Conformal mapping. Prove that $W = z^2$ is conformal.

Q.3(B) Answer any one out of two. (04)

(1) Discuss the transformation w = cos z.

(2) Find the mobious mapping which transforms points z = 1, i, (-1) of z-plane in to w = i, 0, (-i) of w-plane.

Q.4(A) Answer any two out of three. (10)

(1) State and prove Taylor's infinite series for an analytic function.

(2) State and prove Laurent's infinite series for an analytic function.

(3) In usual notation prove that $\cosh(z + z^{-1}) = a_0 + \sum_{n=1}^{\infty} a_n (z^n + z^{-n})$ where $a_n = \frac{1}{2\pi} \int_0^{2\pi} \cosh(2\cos\theta)\cos n\theta \, d\theta.$

Q.4(B) Answer any one out of two. (04)

(1) Expand $z^{-1}(1-z^2)^{-1}$ in Laurent series for $|z| < 1$.

(2) Expand $\frac{z}{(z-1)(z-3)}$ in Laurent's series for $0 < |z-1| < 2$.

Q.5(A) Answer any two out of three. (10)

(1) State and prove Cauchy's residue theorem.

(2) Prove by using Residue Theorem $\int_0^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)} = \frac{\pi}{6}$.

(3) Prove by using Residue Theorem $\int_0^{2\pi} \frac{d\theta}{(a+\cos\theta)^2} = \frac{2\pi a}{\left(\sqrt{a^2-1}\right)^3}$.

Q.5(B) Answer any one out of two. (04)

(1) Discuss the method for finding the residue of f (z) at the z_0 of the order m; $m \geq 2$.

(2) Evaluate $\int_C \frac{e^z dz}{z(z-1)^2}$ dz; C: $|z| = 2$.

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